Mutual Fund Cash Flow Modelling

A model for the balance between the expenses to pay back amortizing notes and the income from fees generated by mutual funds is presented. We provide two basic models, one static and the other dynamic, for the performance of the mutual fund fees. The static model assumes that the Net Asset Value (NAV) of the mutual fund grows at a predetermined rate. The alternative model assumes that the growth rate of the NAV varies either.

The fund manager pays outright commissions to the broker who buys units of the mutual fund to his customers, while the customers do not pay anything at the time of purchase. The fund unit holders, however, pay fees to the fund management annually. The fund sells these future cash flows to one party and uses the proceeds to pay the broker commissions. To finance the purchase of the future cash flows, the party issues amortizing notes.

The fees generated by the mutual fund are of the two kinds. The first are called 12b-1 fees and they are a fixed percentage of the NAV of the fund. The second are redemption fees. These fees are calculated as a percentage of the net asset value of the redeemed shares and this percentage varies with the age of the redeemed shares, typically diminishing to zero over a period of 5-8 years. At the discretion of the fund manager, up to ten percent of the redeemed shares may be exempt from the redemption fee.

As the fund pays dividends, a portion is reinvested into the fund. The shares that are bought on the reinvested dividends can be redeemed without the redemption fees and are therefore called “free shares”.

Fee collection, amortizing payments and interest compounding are made monthly. Dividends are paid and reinvested annually.

Both static and dynamic models take annual schedules of redemption rates, redemption fees, and redemption fee exempt rates as input parameters. NAV/share, dividend yield (called redistribution rate), dividends reinvestment rate, and the amortization period are also input parameters (see https://finpricing.com/lib/IrBasisCurve.html).

The major point of the modeling is to determine whether the cash flow from the mutual fund fees can cover the repayment of the notes (the principal and the interest). In the static model, the actual shortfalls or excess cash flows are put to the output pages. In the dynamic model, the outputs include both indicative and quantitative measures of the excess/loss. According to the Structured Asset Securitization Group, when the dynamic model is used only the indicative output values are considered important.

The static model of the cash flows generated by the fund fees assumes a constant growth rate of the fund’s NAV (which is an input parameter). Assuming that the fund consists of a pool of one-time purchased shares and no dividend reinvestment is made, the number of shares at the end of the n-th month since the purchase date is

$$N_n = N_{n-1}(1 - r_n / 12) \qquad (1)$$

where

- N_n is the number of the fund’s shares at the end of n-th month,

- r_n is an annualized redemption rate for *n*-th month.

The share value grows as

$$P_n = P_{n-1}(1 + g / 12) \tag{2}$$

Where

- P_n a share price at the end of the *n*-th price.
- g is a constant annual growth rate.

This means that, in effect, the share price compounds monthly.

At the end of the n-th month, the NAV, V_n, of the fund becomes

$$V_n = N_n P_n \tag{3}$$

The dynamic model assumes that the growth rate varies from month to month so that

$$P_n = P_{n-1}(1 + g_n / 12) \tag{4}$$

where g_n is an annualized growth rate for *n*-th moth. We also provide a worksheet with the fund's monthly history of the 315 annualized growth rate values for the period from January 1973 to March 1999.

The dynamic model gives an option of using one of four modes of modeling the growth rate: one deterministic and three stochastic modes. In each mode, the calculations of values described below are repeated a specified number of times and the results are statistically processed.

In the deterministic mode, a sliding window of the length of the notes amortization period (70 month long in the considered case) is applied to the history of the fund's growth rate starting at the beginning. The sequence of the growth rate within the window is taken as the rate sequence during the notes amortization period. For each subsequent run the window is moved forward one month forward.

In a stochastic mode (called "Just start") the same window is applied each run; however, each time the position of the window is chosen randomly (with the uniform distribution over the history array).

In another stochastic mode (called "All 96 Points") the growth rate for each of the 70 months within the modeled period is picked randomly, also with the uniform distribution, from the historical series of 315 values. Note that the number "96" in the name of the mode does not have any direct meaning, as in this case 70 points are actually picked from the history array.

Finally, in the mode called "Monte Carlo", the growth rate is made a normally distributed random variable. Its mean and standard deviations are input parameters.

Once the sequence of growth rates are formed according to the chosen mode, we calculate for each period the number of the fund's shares, the NAV per share, the collected fee, the interest due on the notes, the sum of all fees by the end of the amortization period and, finally, whether the notes are paid off by the end of the amortization period. In addition, the values "Frequency of Loss" and "Frequency of Timely Payments" are formed as follows:

$$Frequency\ of\ loss = \begin{cases} 0 & \text{if notes are paid off} \\ 1 & \text{if the run results in a loss} \end{cases}$$

$$Frequency\ of\ timely\ payments = \frac{number\ of\ periods\ with\ timely\ payments}{total\ number\ of\ periods}$$

After the specified number of runs are completed, the absolute maximum, absolute minimum, average, median, and standard deviations for each value are calculated.